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Rule-Based Mamdani-Type Fuzzy Modeling of Perceived Stress, And Cortisol Responses to Awakening

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ABSTRACT
In this paper, Two Mamdani type fuzzy models (four inputs–one output and two inputs–one output) were developed to test the hypothesis that high job demands and low job control (job strain) are associated with elevated free cortisol levels early in the working day and with reduced variability across the day and to evaluate the contribution of anger expression to this pattern. The models were derived from multiple data sources including One hundred five school teachers (41 men and 64 women) classified 12 months earlier as high (N = 48) or low (N = 57) in job strain according to the demand/control model sampled saliva at 2-hour intervals from 8:00 to 8:30 hours to 22:00 to 22:30 hours on a working day. The quality of the model was determined by comparing predicted and actual fuzzy classification and defuzzification of the predicted outputs to get crisp values for correlating estimates with published values. A modified form of the Hamming distance measure is proposed to compare predicted and actual fuzzy classification. An entropy measure is used to describe the ambiguity associated with the predicted fuzzy outputs. The four input model predicted over 70% of the test data within one-half of a fuzzy class of the published data. The two input model predicted over 40% of the test data within one-half of a fuzzy class of the published data. Comparison of the models show that the four input model exhibited less entropy than the two input model.
Key words: job strain, cortisol, anger, work stress, teaching, Mamdani fuzzy modeling, Hamming distance
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I. INTRODUCTION

Work stress has emerged as a major psychosocial influence on physical and mental health over recent decades. Fuzzy model has proved valuable in understanding the work characteristics associated with coronary heart disease risk, hypertension, mental health, quality of life, and other outcomes. This model proposes that people working in highly demanding jobs who also have low control and limited opportunities to use skills will experience high job strain. The HPA axis is one of the principal pathways activated as part of the physiological stress response. It is puzzling, therefore, that clear links between cortisol and work stress has not been established. This study used serial sampling over the working day to determine whether salivary cortisol was predicted by measures of job strain taken 1 year earlier in men and women working in a single occupation. Several approaches to the investigation of cortisol and work stress have been taken. Cross-sectional studies with a single saliva or blood sample have generally not shown consistent associations with different aspects of work stress, and in two studies, cortisol was lower in individuals reporting high job strain. Luecken et al. [1] found that cortisol excretion during the working day in a sample of working women was not related to demands, control, or strain at work but was greater among those reporting high strain at home due to domestic responsibilities.

This raises the possibility that the inconclusive effects observed in previous studies have arisen with cortisol assessed later in the day. However, it has also been proposed that the diurnal rhythm of cortisol output is disturbed with chronic stress and that maintenance of neuroendocrine activation after termination of stressful stimulation may characterize chronic allostatic load, leading to a "flat" circadian rhythm [2], [3], [4]. We therefore obtained measures of salivary cortisol throughout the working day and evening and assessed differences between day and evening as well as early morning levels. We hypothesized that job strain would be associated with elevated cortisol early in the morning together with heightened cortisol later in the day. Such a pattern might lead to reduced variability in cortisol output over the working day. An additional aim of this study was to investigate possible interactions between job strain and anger expression. Hostility and cynical distrust have been associated with

increased risk of coronary heart disease and the progression of carotid atherosclerosis [5][6][7]. The inhibition of anger expression has also been related to atheroma [8].

This paper presents a Mamdani fuzzy modeling scheme where rules are derived from multiple knowledge sources such as previously published databases and models, existing literature, intuition and solicitation of expert opinion to verify the gathered information. The output or consequence of a Mamdani-type model is represented by a fuzzy set. To assess model performance, a crisp estimate of the consequence is usually made by defuzzification methods such as the centroid, weighted average, maximum membership principle and mean membership principle [9]. Depending on the shape of the output fuzzy set, defuzzification methods do not effectively characterize the output with the corresponding ambiguity associated with the prediction. An alternative strategy could be implemented such that the actual values of the output infer an ordinal set representing a three point fuzzy classification (low and high) that could be compared to the actual fuzzy classification using distance measures. In addition, the ambiguity associated with the predicted fuzzy sets can be quantified by calculating entropy [10].

The purpose of this study was to develop generalized rule based fuzzy models from multiple knowledge sources to test the hypothesis that high job demands and low job control (job strain) are associated with elevated free cortisol levels early in the working day and with reduced variability across the day and to evaluate the contribution of anger expression to this pattern and subsequently test its performance by comparing defuzzified outputs to actual values from test data and comparing predicted and actual fuzzy classifications. The overall approach followed in this study is illustrated in Figure 1. The process begins with knowledge acquisition, continues to model building and then finally testing the model performance. In the context of fuzzy modeling, the proposed approach of converting the predicted fuzzy output and the actual crisp value into fuzzy classification sets is not well defined in literature.

II. MAMDANI-TYPE FUZZY MODELING

As the complexity of a system increases, the utility of fuzzy logic as a modeling tool increases. For very complex systems, few numerical data may exist and only ambiguous and imprecise information and knowledge is available. Fuzzy logic allows approximate interpolation between input and output situations. Two main types of fuzzy modeling schemes are the Takagi–Sugeno model and the fuzzy relational model. The Takagi–Sugeno scheme is a data driven approach where membership functions and rules are developed using a training data set. The parameters for the membership functions and rules are subsequently optimized to reduce training error. The relationship in each rule is represented by a localized linear function [11]. The final output is a weighted average of a set of crisp values. The Mamdani scheme is a type of fuzzy relational model where each rule is represented by an IF– THEN relationship. It is also called a linguistic model because both the antecedent and the consequent are fuzzy propositions [11]. The model structure is manually developed and the final model is neither trained nor optimized. The output from a Mamdani model is a fuzzy membership function based on the rules created. Since this approach is not exclusively reliant on a data set, with sufficient expertise on the system involved, a generalized model for effective future predictions can be obtained. Consider a simple two input–one output Mamdani type fuzzy model. The rule structure is represented in Figure

Each row of membership functions constitutes an IF– THEN rule, also defined by the user. Depending on the values used, the input membership functions are activated to a certain degree. The contributed output from each rule reflects this degree of activation. The final output is a fuzzy set created by the superposition of individual rule actions (Figure 1).
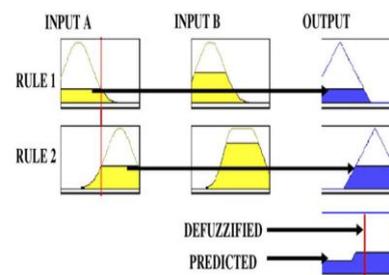
Figure1: Examples of a Mamdani type fuzzy inference system

2.1. DEFUZZIFICATION METHODS

The fuzzy output is obtained from aggregating the outputs from the firing of the rules. Subsequent defuzzification methods on the fuzzy output produce a crisp value. Two common techniques for defuzzification are the maxima methods and area-based methods,

which are briefly explained. Several such methods are explained by Ross.

2.1.1. MAXIMA METHODS

The maxima methods identify the locations where maximum membership occurs. Either one such point is selected as the defuzzified value (Figure 2A) or an average of all points with maximum membership is selected as the crisp value (Figure 2B). The advantages of the maxima methods are their simplicity and speed. The major disadvantage is loss of information as only rules of maximum activation are considered.

2.1.2. AREA-BASED METHODS

A popular area-based defuzzification procedure is the centroid method. As the term implies, the point of the output membership function that splits the area in half is selected as the crisp value (Figure 2C). This method however does not work when the output membership function has non-convex properties. Depending on the shape of the membership function of the output, defuzzification routines may not produce effective values for the predicted output. However, the defuzzified value using the mean-max membership principle that does not convey the ambiguity. The centroid method has drawbacks when the output membership function is non-convex (Figure 2B). The defuzzified value is at a point that has low membership. In an effort to compensate for these drawbacks, an alternative approach to model validation is proposed that uses a distance measure to compare actual and predicted fuzzy classifications consisting of three point ordinal sets.
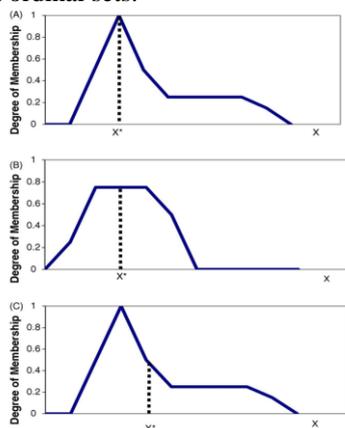
Figure 2. Different defuzzification methods: (A) max-membership principle; (B) mean-max membership principle; (C) centroid principle. Note: *x** is the defuzzified value.

2.2. DISTANCE MEASURES BETWEEN FUZZY SETS

For two fuzzy sets A and B in the same universe, the Hamming distance [12] is an ordinal measure of dissimilarity.
The Hamming distance (HD) is defined as:

$$HD(A,B) = \sum_{i=1}^{n} \left| \mu_A(x_i) - \mu_B(x_i) \right| \qquad (1)$$

where n is the number of points that define the fuzzy sets A and B, $\mu_A(x_i)$ the membership of point x_i in A and $\mu_B(x_i)$ is the membership of point x_i in B. The Hamming distance is smaller for fuzzy sets that are more alike than those that are less similar. In comparing an actual fuzzy set to the predicted fuzzy set, a small Hamming distance is ideal. In our study, the model-testing phase involved comparison of predicted and actual fuzzy classifications (low and high). From the results in Table 1, the proposed distance measure is better than the Hamming distance at distinguishing between different levels of classification. In cases e and f, the Hamming distance(HD) gave the same value for different predicted fuzzy classifications. The proposed modified Hamming distance gave different values that effectively distinguish between these cases.

2.3. ENTROPY OF A FUZZY SET

Entropy is a measure of fuzziness associated with a fuzzy set. The degree of fuzziness can be described in terms of a lack of distinction between a fuzzy set and its complement. For a fuzzy set A, entropy [7] is calculated as:

$$E(A) = 1 - \frac{1}{n} \sum_{i=1}^{n} \left| 2\mu_A(x_i) - 1 \right| \qquad (2)$$

where n is the number of points that define A, and $\mu_A(x_i)$ is the membership of point x_i in A. In this study, the concept of entropy was used to quantify the ambiguity associated with the predicted fuzzy outputs. In the absence of actual values, entropy values are essentially a measure of confidence in outputs predicted by a fuzzy model.

2.4. PROPOSED DISTANCE MEASURE

As indicated in the theory section, a modified form of the Hamming distance is proposed which enables better distinction between different levels of classification (see Table1 and 2).

The proposed distance measure D(A, P) is defined as:

$$D(A,B)= \frac{1}{4}(\sum_{i=1}^{n}|\mu_A(x_i)-\mu_B(x_i)| + \sum_{i,k=1(i\neq k)}^{n}(2(2|i-k|-1)\mu_A(x_i)\mu_P(x_k))) \qquad (3)$$

where A is the actual fuzzy classification, P the predicted fuzzy classification, n the number of classes that define A and P, $\mu_A(x_i)$ is the membership of point x_i in A and $\mu_P(x_k)$ is the membership of point x_k in P.

III. COMPARING FUZZY CLASSIFICATIONS

The two output membership functions created in both models are categorized as low and high. The actual value from the test data was evaluated using the parameters of these membership functions to produce a fuzzy set represented by two points (high and low). This fuzzy set represents the degree of belongingness (μ) to each of the two categories (low and high). The predicted output from the Mamdani model is a fuzzy set represented by the given points. Based on the relative contributions from each output membership function, the predicted fuzzy set of given points was reduced to a fuzzy set of three points. The relative contributions from each output membership function were estimated by integrating the predicted fuzzy set over the range of the membership function. Equations (4) were used to develop the predicted fuzzy classification:

$$\mu_P(x) = \int f(x).\mu_i(x)dx / \int \mu_i(x)dx \qquad \dots (4)$$

For each test case, an actual fuzzy classification and a predicted fuzzy classification were obtained. The modified Hamming distance measure (3) was used to determine the similarity between the two fuzzy sets. Apart from a comparison to actual values, the ambiguity associated with each predicted value was quantified using an entropy measure (2) as defined in the theory section.

3.1. DEFUZZIFYING THE PREDICTED OUTPUT

The centroid method was used to defuzzify the output of the Mamdani models. The crisp predictions were compared to the actual values from the test data and entropy value was calculated. This is a common form of comparison utilized for most modeling strategies. However, defuzzifying the output results in a loss of information regarding the ambiguity of the prediction. In the absence of actual values, the confidence in the prediction

can be determined based on the degree of ambiguity.

3.2. COMPARING PREDICTED AND ACTUAL FUZZY CLASSIFICATION

During the testing of each model, fuzzy classifications were created for the predicted and actual values using defined output membership functions. Each fuzzy classification set was represented by three membership values: high and low. The proposed distance formula was applied in each test case and an estimate of classification was obtained. The distribution of the calculated distances for both models is provided in Figure 6. A distance measure of two implies that the model prediction was two fuzzy classes from the actual value. In both models, all the test data was predicted within one fuzzy class of the actual value. However, the performance of the three input model does appear to be significantly better.

IV. EXAMPLE

Data were collected at the 12-month follow-up phase of a study of job strain and cardiovascular risk, details of which have been published previously [13]. Participants in the original sample were 162 junior and high school teachers, selected on the basis of scores on a work stress measure (37) as having high (28 men and 52 women) or low (32 men and 50 women) job strain scores. Eighty-five (52.5%) were classroom teachers, and 77 (47.5%) had additional administrative roles. One hundred thirty-seven teachers took part in the 12-month phase (84.6%), which consisted of ambulatory blood pressure monitoring and a psychiatric interview (to be reported elsewhere) in addition to cortisol measurements. Of the 25 who did not participate at 12 months, 10 had left teaching or retired, 7 were seriously ill or pregnant, 1 experienced equipment failure, and 7 did not respond to our invitation. Comparisons between the 137 participants and 25 who dropped out of the study revealed no significant differences in gender, job strain scores, age, grade of employment, or scores on negative affect or anger expression. An additional 15 of the 137 individuals refused to sample saliva during the working day, mainly because they envisaged that data collection might be embarrassing or inconvenient at school. Statistical comparisons of these individuals with the remainder again identified no differences on demographic or psychological variables.
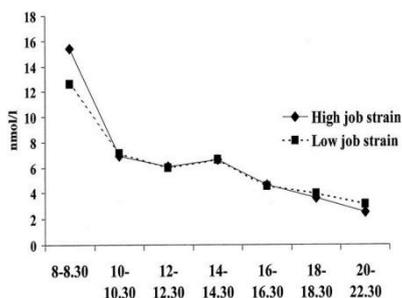
Figure 3. Mean concentration of saliva free

Fuzzy function of the given figure 3 and 5 is defined as

$$f(x) = \begin{cases} -5x + 1.5, & x \in [0, 0.2] \\ 0.5, & x \in [0.2, 0.4] \\ -x + 0.9, & x \in [0.4, 0.7] \end{cases}$$

$$A_1(x) = \begin{cases} 5x, & x \in [0, 0.2] \\ 2 - 5x, & x \in [0.2, 0.4] \\ 0, & otherwise \end{cases}$$

$$A_2(x) = \begin{cases} 5x - 1, & x \in [0.2, 0.4] \\ -3.33x + 0.33, & x \in [0.4, 0.7] \\ 0, & otherwise \end{cases}$$

Corresponding Fuzzy diagram of given figure 3 and figure 5
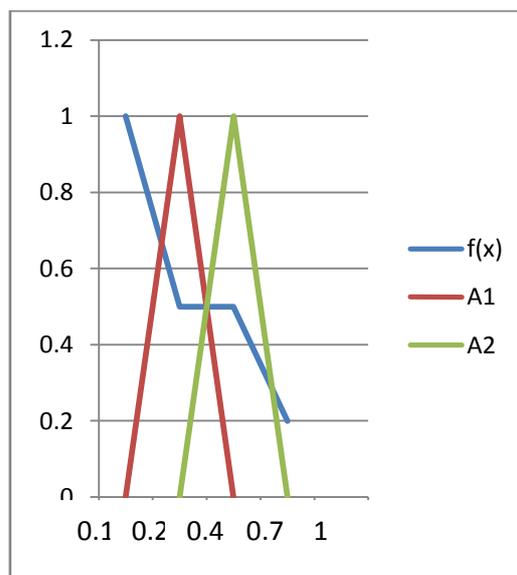
Figure 4. Fuzzy Mean concentration of saliva free cortisol in high and low job strain groups across the day and evening and Fuzzy Mean concentration of saliva free cortisol in men and women across the day and evening.

Case		Actual fuzzy classification		Predicted fuzzy classification		HD	Predicted Distance
	Time	High	Low	High	Low		
a	8 – 8.30	1	0.83	0.9	0.8	0.13	0.43
b	10 – 10.30	0.45	0.45	0.8	0.8	0.7	0.35
c	12 – 12.30	0.39	0.39	0.5	0.5	0.22	0.15
d	14 – 14.30	0.44	0.44	0.4	0.4	0.08	0.11
e	16 – 16.30	0.31	0.31	0.5	0.5	0.38	0.17
f	18 – 18.30	0.24	0.27	0.4	0.5	0.39	0.16
g	20 – 22.30	0.17	0.21	0.4	0.5	0.52	0.17
Entropy Value		0.57	0.64	0.71	0.8		

Table 1: Comparison of the Hamming distance and the proposed modified Hamming distance of Fuzzy Mean concentration of saliva free cortisol in high and low job strain groups across the day and evening

Three items indexed job demands (eg, "The pace of work in my job is very intense"), three items concerned perceived job control (eg, "I have freedom to decide what I do in my job"), and four related to skill utilization (eg, "My job involves me in learning new things"). Each item was rated on a four-point scale ranging from "strongly disagree" to "strongly agree." Scores for each dimension were scaled to range from 0 to a maximum of 10. An index of job strain was derived, where job strain = job demands/(job control + skill utilization). A job strain of 10 indicates a perfect balance between demands and control, with higher scores reflecting high demand coupled with low control and skill utilization.
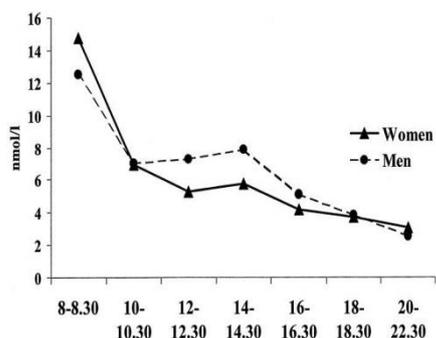
Figure 5. Mean concentration of saliva free cortisol in men and women across the day and evening.

Saliva sampling was conducted on a working day at schools. Participants were asked to take eight saliva samples at 2-hour intervals, and a 30-minute time window was allowed for each sample. Participants were asked to not consume any caffeine, citrus drinks, or food for at least 60 minutes before the saliva sample was taken. The schedule sampling sequence was therefore 8:00 to 8:30, 10:00 to 10:30, 12:00 to 12:30, 14:00 to 14:30, 16:00 to 16:30, 18:00 to 18:30, 20:00 to 20:30, and 22:00 to 22:30 hours. The first sample of the day was always obtained at schools after explanation of the procedure by the investigators. Saliva samples were collected in Salivettes, which were stored at −30°C until analysis. After defrosting, samples were spun at 3000 rpm for 5 minutes, and 100 μl of supernatant was used for duplicate analysis involving a time-resolved immunoassay with fluorescence detection.

Case		Actual fuzzy classification		Predicted fuzzy classification		HD	Predicted Distance
	Time	Women	Men	Women	Men		
a	8 – 8.30	1	0.83	0.9	0.8	0.13	0.43
b	10 – 10.30	0.6	0.6	0.8	0.8	0.4	0.34
c	12 – 12.30	0.34	0.5	0.5	0.5	0.16	0.13
d	14 – 14.30	0.37	0.53	0.4	0.4	0.16	0.11
e	16 – 16.30	0.27	0.33	0.5	0.5	0.4	0.18
f	18 – 18.30	0.25	0.25	0.4	0.5	0.4	0.16
g	20 – 22.30	0.2	0.19	0.4	0.5	0.51	0.18
Entropy Value		0.52	0.66	0.71	0.8		

Table 2: Comparison of the Hamming distance and the proposed modified Hamming distance of Fuzzy Mean concentration of saliva free cortisol in men and women across the day and evening.
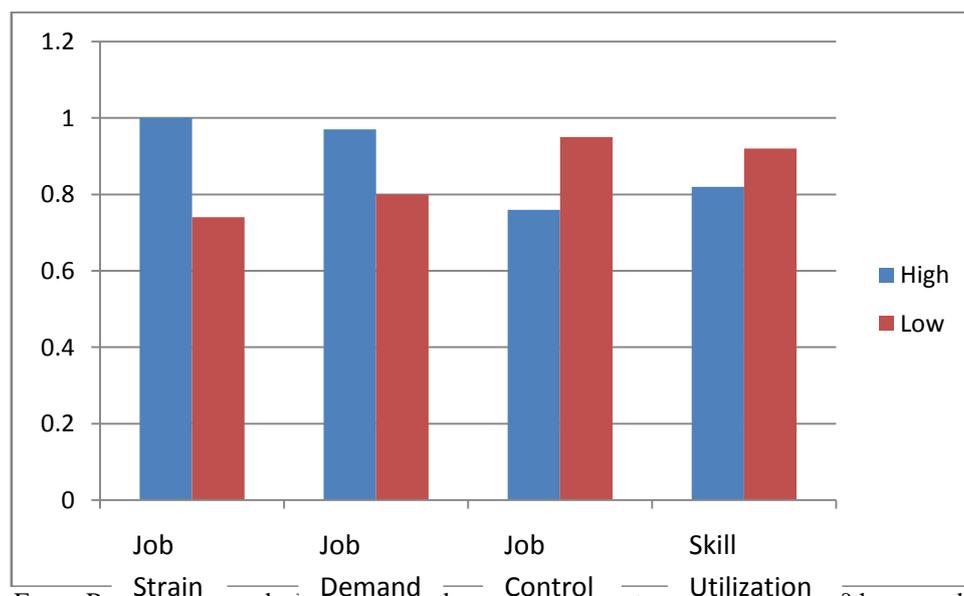
Figure 6: Fuzzy Re[...]an dec[...]l co[...]etwe[...]0 hours and 20:00 and 22:30 hours, adjusted for age and negative affect, in participants divided on the basis of job strain, job demands, job control, and skill utilization.

V. CONCLUSIONS

Job strain is associated with elevated free cortisol concentrations early in the working day but not with reduced cortisol variability. The interaction with outward anger expression suggests that individual characteristics modulate the impact of chronic work stress on the hypothalamic-pituitary-adrenocortical system. Two Mamdani-type models were developed to predict characteristics of the sample are summarized in figure 3 and 5. The high and low job strain groups did not differ in gender distribution, age, occupational grade, or proportion of cigarette smokers. There were significant differences between groups in job strain and in its components job demands, job control, and skill utilization. The high job strain group reported greater demands, lower control, and less skill utilization than the low job strain group as inputs. Negative affect was significantly higher among high job strain individuals, and anger-in scores were also greater. There were no differences in anger-out ratings between groups. Using multiple knowledge sources, membership functions and rules were developed to provide generalized models not optimized for a specific data set. Apart from correlation estimates of actual and defuzzified predictions, an alternative analysis was performed involving comparison of actual and predicted fuzzy classifications. A distance measure was used to compare actual and fuzzy classifications. The proposed measure is a modification of the Hamming distance often used to compare distances between fuzzy sets. The Mamdani model developed is a knowledge-driven predictive model that is not common in Job strain and in its components job demands, job control, and skill utilization. A major advantage of this modeling approach is that it enables the use of entropy measures to quantify ambiguity associated with future predictions.

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